Thermally Induced Scattering of Light in Fine-Grained Ceramics

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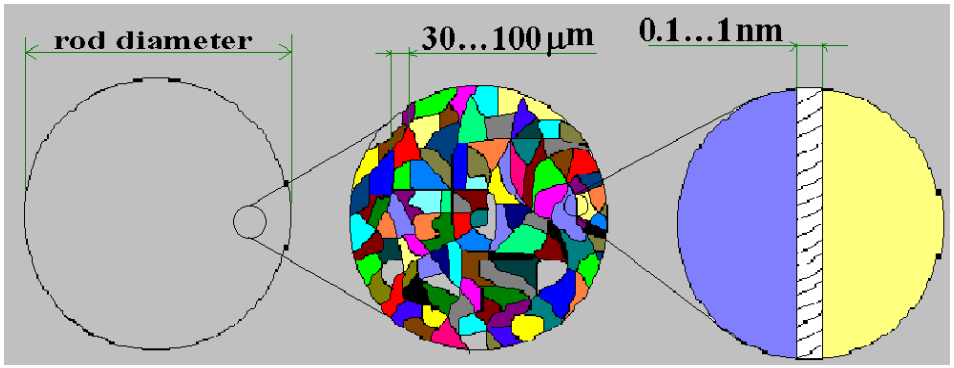
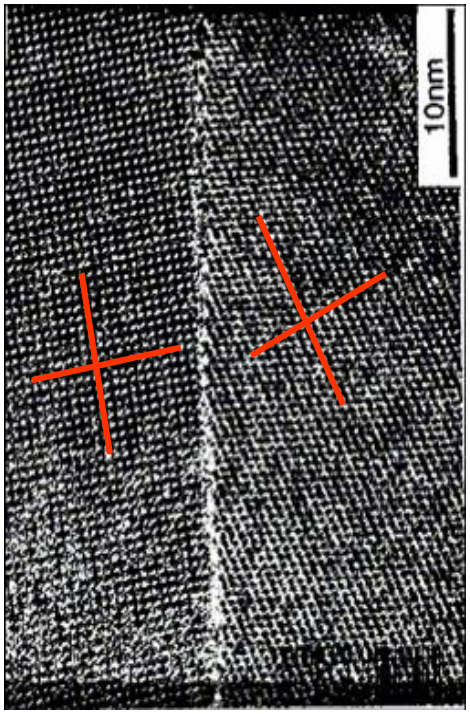
Outline

- Introduction
- Thermally induced scattering of light for the unconditioned grain size
- Comparison of the results with the geometric optics calculation
- Conclusion

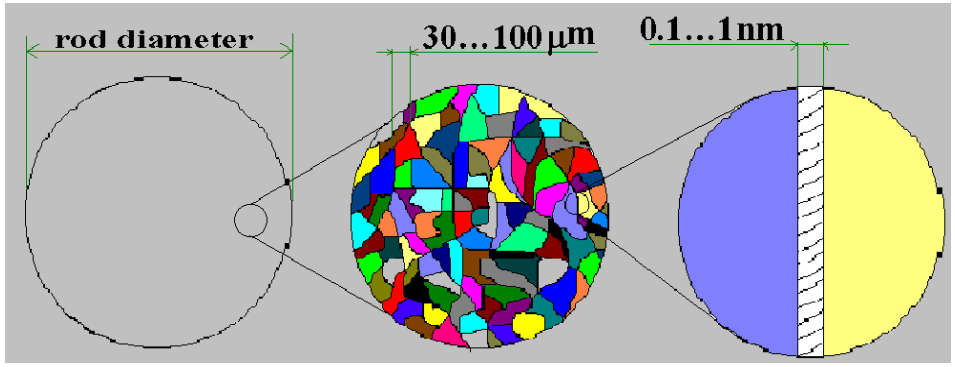
Introduction

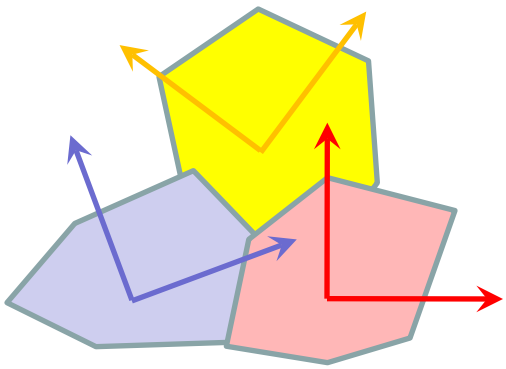
- The advantages of laser ceramics as a laser material
  - Large apertures of the optical elements (like glasses)
  - High thermal conductivity (at room temperatures like single crystals)
  - Ceramics is similar to single crystals in the absorption and emission spectra as well as the fluorescence lifetime





Introduction





The specific property of laser ceramics is the thermally induced distortion of the transmitting beam due to grains

- Laser ceramics is usually made of cubic crystals and is naturally optically isotropic
- The absorption of radiation leads to heating of the optical elements
- Temperature gradients induce elastic stresses
- Photoelastic effect induces optical anisotropy in the medium
- Crystallographic axes variation from grain to grain results in the dielectric permeability tensor variation

the dielectric impermeability tensor

$$\mathbf{B} = \boldsymbol{\varepsilon}^{-1}$$

the dielectric permeability tensor

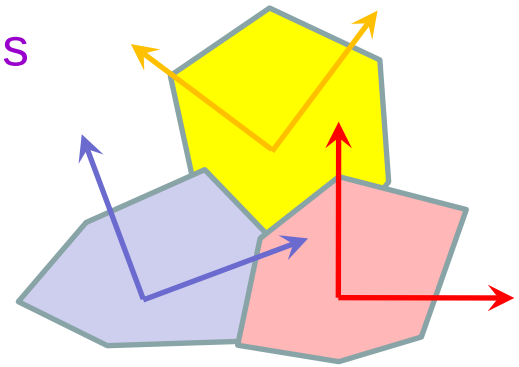
$$\boxed{\Delta B_{ij} = \pi_{ijkl}\sigma_{kl}}$$

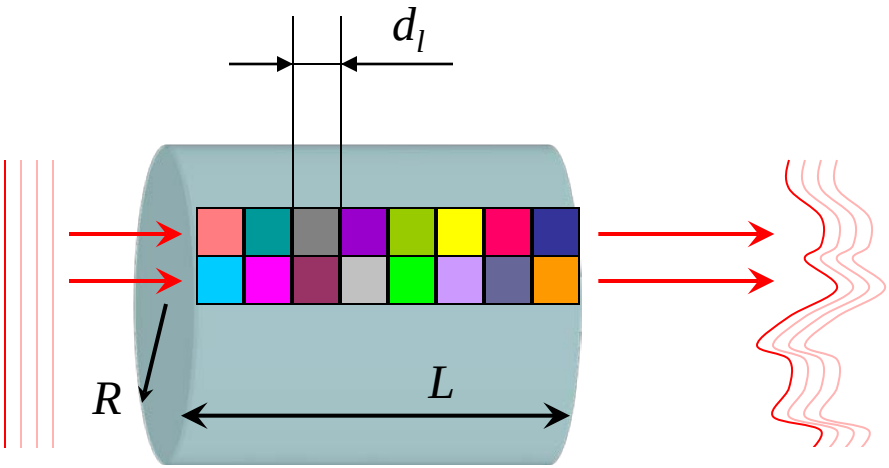
the piezooptic tensor

the stress tensor

Thermally induced beam distortions in the large grain size case

- For each geometrical optics ray laser ceramics is treated as a series of phase plates
- Each plate is birefringent and has its own eigen polarizations and eigen phase shifts





Phase variation

[1] Snetkov, I. L., Mukhin, I. B., Palashov, O. V., and Khazanov, E. A., Quantum Electron. 37(7) 633–638 (2007).

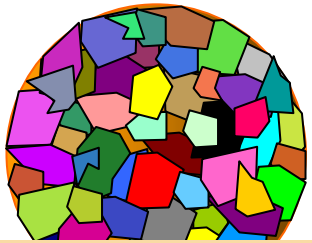
Polarization distortion

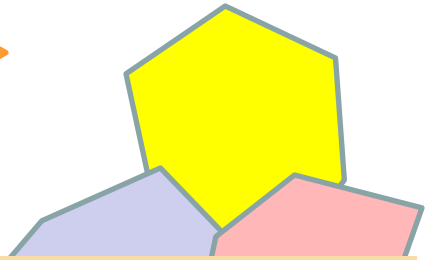
[2] Kagan, M. A. and Khazanov, E. A., Quantum Electron. 33(10), 876–882 (2003).

Distortions of the transmitted beam in ceramics with large and small grains

Small grains

Large grains





More grain boundaries and related “cold” distortions

Higher level of thermally induced distortions

- Modern technologies allow fabrication of ceramics with thin grain boundaries (< 1 nm) giving the opportunity to reduce grain size (up to ~ 1 μm) keeping optical quality of the samples and reducing the thermally induced distortions
- A new calculation of thermal effects is required when the grain size is smaller than the wavelength since the geometrical optics approximation becomes inaccurate

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Beam distortions for the unconditioned grain size

$$\varepsilon_{ij} = \left(\varepsilon_0 + \varepsilon_T\right)\delta_{ij} + \varepsilon_{A\,ij} + \varepsilon_{1\,ij}$$

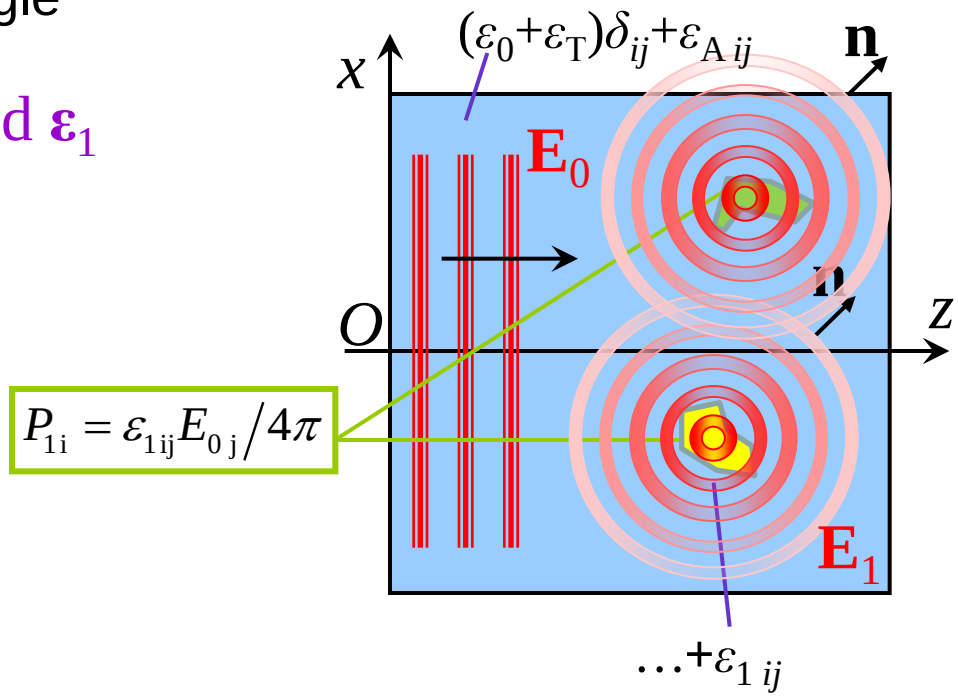
the "cold" dielectric constant

the thermal additive

the average photoelastic additive

the fluctuative photoelastic additive

- We used the Debye-Rayleigh scattering theory (the small disturbances method) in single scattering approximation
- We assumed that ε_T, $\boldsymbol{\varepsilon}_A$, and $\boldsymbol{\varepsilon}_1$ are small



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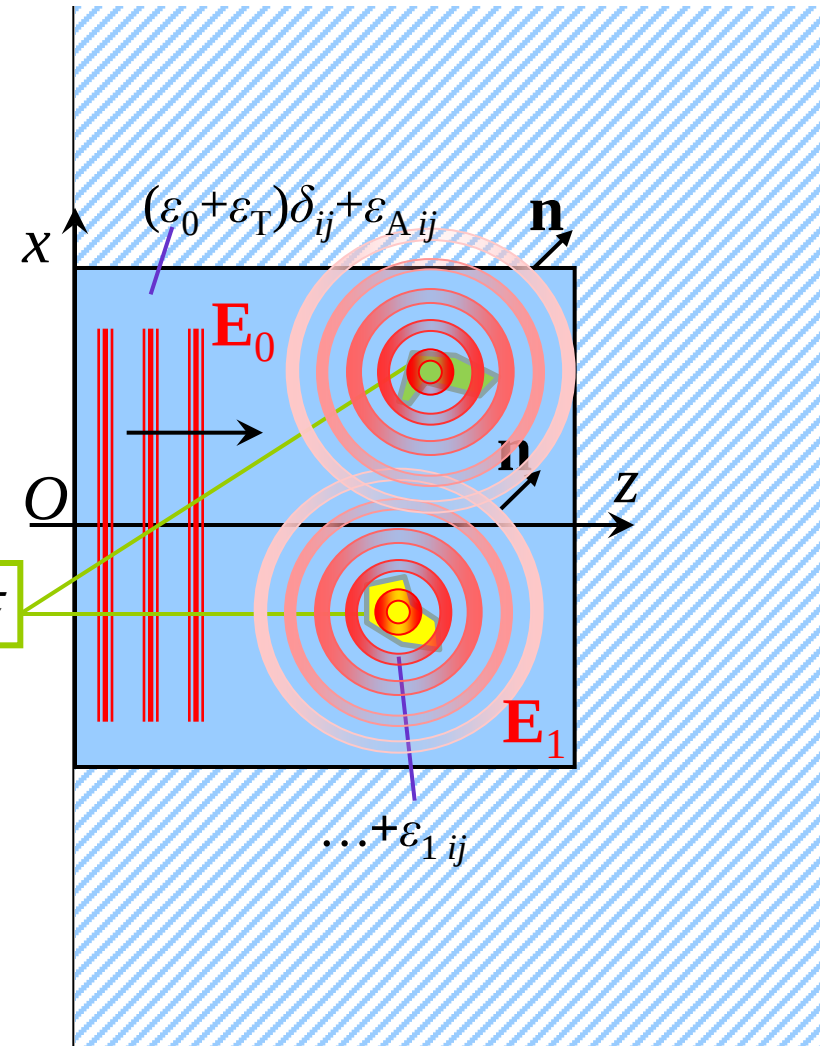
- We used the Debye-Rayleigh scattering theory (the small disturbances method) in single scattering approximation
- We assumed that ε_T, $\boldsymbol{\varepsilon}_A$, and $\boldsymbol{\varepsilon}_1$ are small
- The transferred field can be treated as a power series of $\boldsymbol{\varepsilon}_1$:

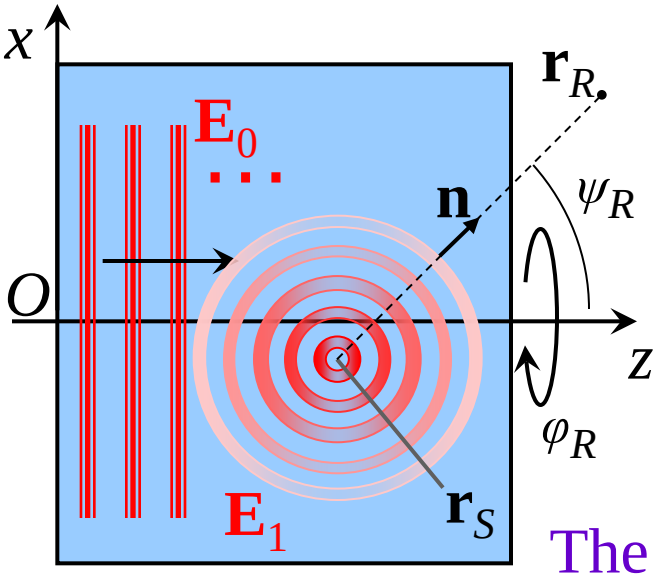
$$\mathbf{E}=\mathbf{E}_0+\mathbf{E}_1+\cancel{\ldots}, \qquad \mathbf{E}_i \sim \varepsilon_1^{\,i}$$

- Reflection and refraction of radiation on the ceramic faces as well as the attenuation of $\mathbf{E}_0$ must be manually treated

$$P_{1i} = \varepsilon_{1ij} E_{0j} / 4\pi$$



Beam distortions for the unconditioned grain size



The average scattered intensity:

$$\left\langle \mathbf{S}_{1\infty}(\mathbf{r}_R)\right\rangle = S_0 \frac{\pi}{2} k_0^{\,4} \frac{\mathbf{n}}{\mathbf{r}_R^{\,2}} \Phi_0(q_0) \iiint_V f_E^{\,2}(r_S) K_\Sigma(\mathbf{r}_S)\, d^3\mathbf{r}_S$$

a spherical wave — the direction pattern

$K_\Sigma(\mathbf{r})$ is a superposition of the correlations of various $\boldsymbol{\varepsilon}_1$ components and the dipole direction pattern

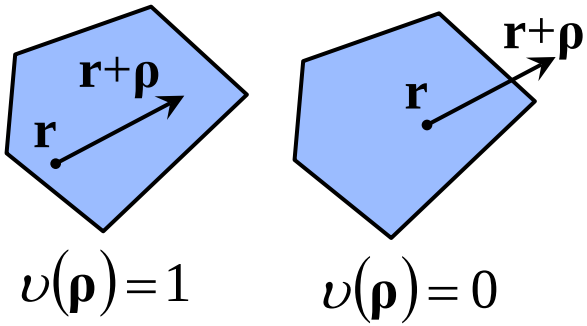
The extinction coefficient (the attenuation factor of $\mathbf{E}_0$):

$$\left\langle h(\mathbf{r}_S)\right\rangle = 4\pi^2 k_0^{\,4}\left[K_{\Sigma 1}(\mathbf{r}_S)\left(I_{\Phi 1} - 2I_{\Phi 2}\right) + 4K_{\Sigma 2}(\mathbf{r}_S) I_{\Phi 2}\right]$$

$$I_{\Phi 1} = \int_0^1 \tau \Phi_0\left[2k_0\sqrt{\varepsilon_0}\,\tau\right] d\tau, \quad I_{\Phi 2} = \int_0^1 \left(\tau^3 - \tau^5\right)\Phi_0\left[2k_0\sqrt{\varepsilon_0}\,\tau\right] d\tau.$$

the usability condition:

$$\boxed{\left\langle h(\mathbf{r}_S)\right\rangle << 1/L}$$



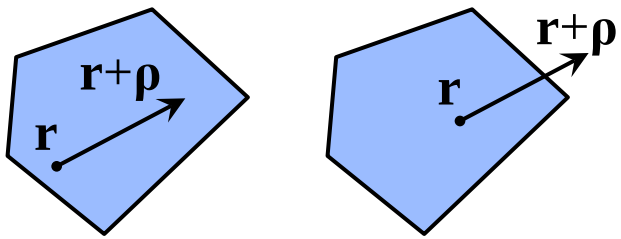
The spatial spectrum of correlation

$$\Phi_0(\kappa) = \frac{1}{2\pi^2\kappa}\int_0^\infty \left\langle \upsilon(\rho)\right\rangle \sin(\kappa\rho)\,\rho\, d\rho$$

The vector of scattering

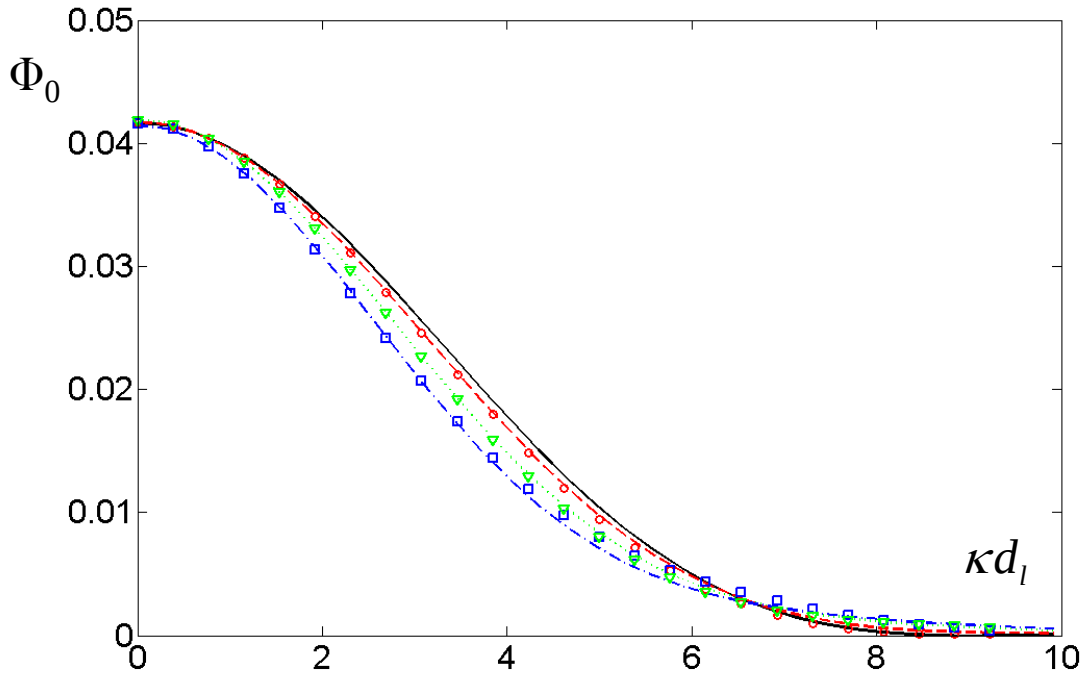
$$\mathbf{q}_0 = k_0 \varepsilon_0^{\,1/2}(\mathbf{z}^0 - \mathbf{n}) \qquad q_0 = 2k_0\varepsilon_0^{\,1/2}\sin(\psi_R/2)$$

The spatial spectrum of correlation

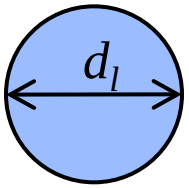


$\upsilon(\boldsymbol{\rho})=1$ $\upsilon(\boldsymbol{\rho})=0$

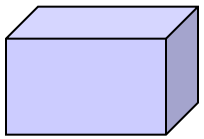
$$\Phi_0(\kappa)=\frac{1}{2\pi^2\kappa}\int_0^\infty\langle\upsilon(\rho)\rangle\sin(\kappa\rho)\,\rho\,d\rho$$



1) solid curve

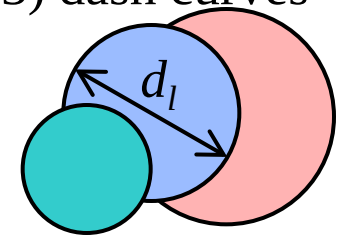


2) symbols



$d_1\times d_2\times d_3$

3) dash curves



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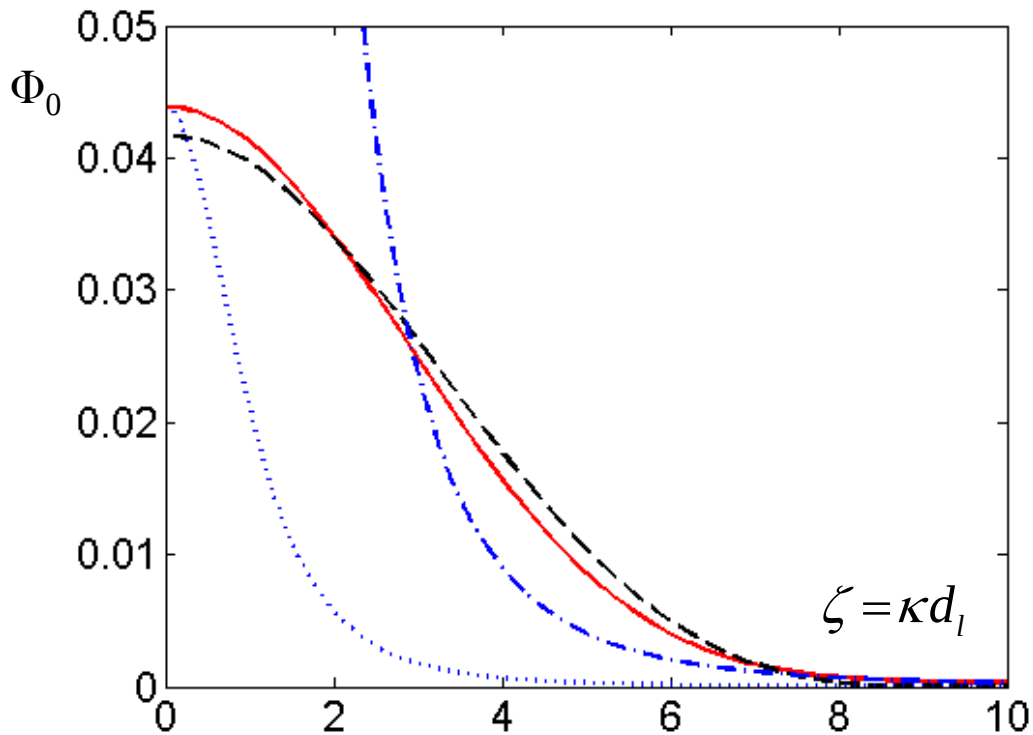
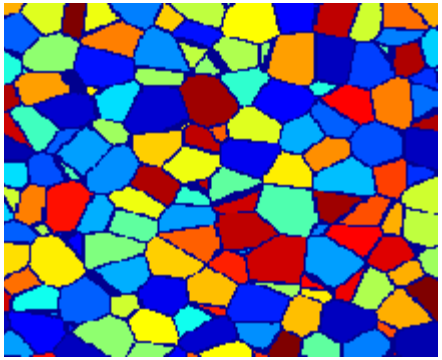
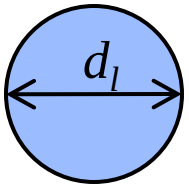
$$\Phi_0(\kappa) = \frac{1}{2\pi^2\kappa}\int_0^\infty \langle \upsilon(\rho)\rangle \sin(\kappa\rho)\,\rho\, d\rho$$



2) numeric modeling (3D Voronoi diagram)



1) Spherical single-sized grains



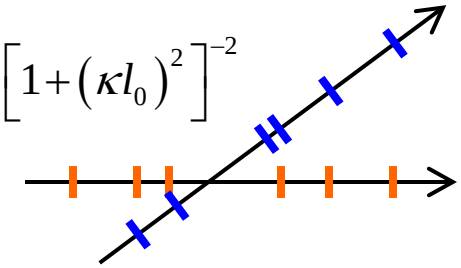
$$\Phi_0(\kappa) = \frac{3d_l^{\,3}}{\pi^2\zeta^4}\left(\cos\frac{\zeta}{2} - \frac{2}{\zeta}\sin\frac{\zeta}{2}\right)^2$$

$$\zeta = \kappa d_l$$

3) The Markovian process (R. V. Jones)

$$\Phi_0(\kappa) = l_0^{\,3}\pi^{-2}\left[1+(\kappa l_0)^2\right]^{-2}$$

$$l_0 = \tfrac{2}{3}\, d_l$$



The direction pattern of the scattered radiation

- Uniform heating
- Flat-top probing beam of radius R

$$p \ll 1:$$

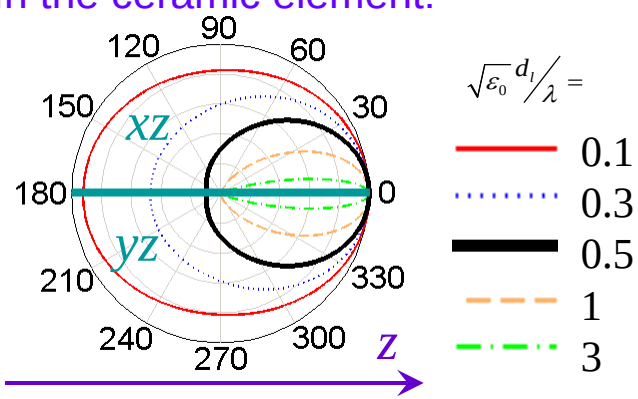
$$D_\Sigma \propto \Phi_0(q_0)\left\{6-\sin^2\psi_R\left[1-(\cos 2\varphi_R)/30\right]\right\}$$

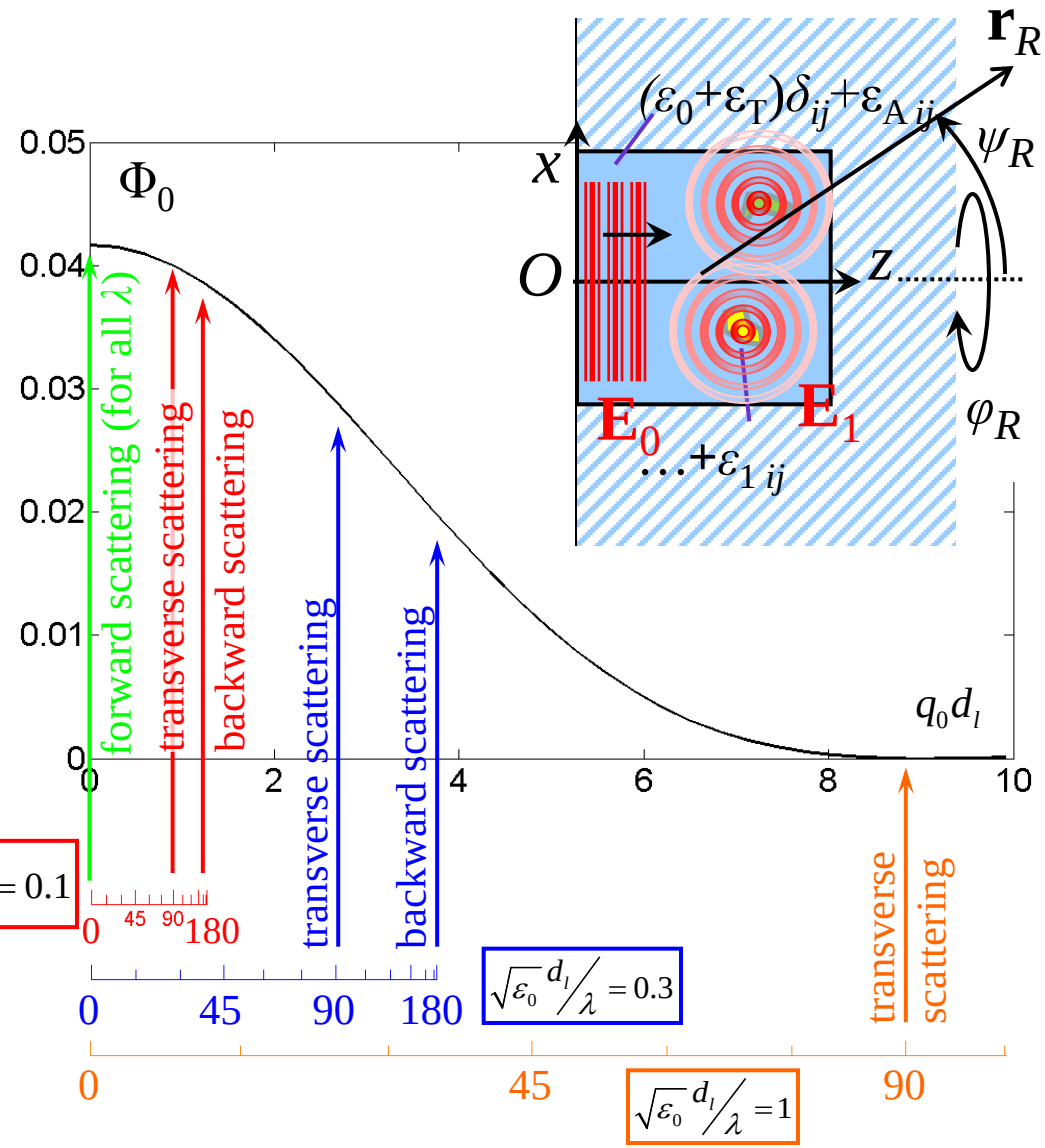
$$p \gg 1:$$

$$D_\Sigma \propto \Phi_0(q_0)\left\{6-\sin^2\psi_R\left[1-(\cos 2\varphi_R)/6\right]\right\}$$

$$q_0 = 2k_0\varepsilon_0^{1/2}\sin(\psi_R/2)$$

In the ceramic element:





corresponding ψ angles for various wavelengths

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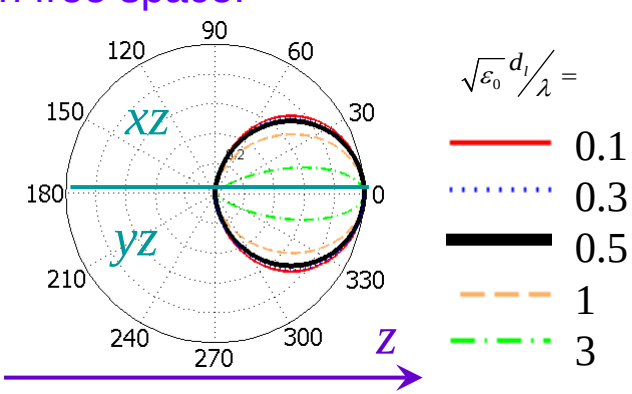
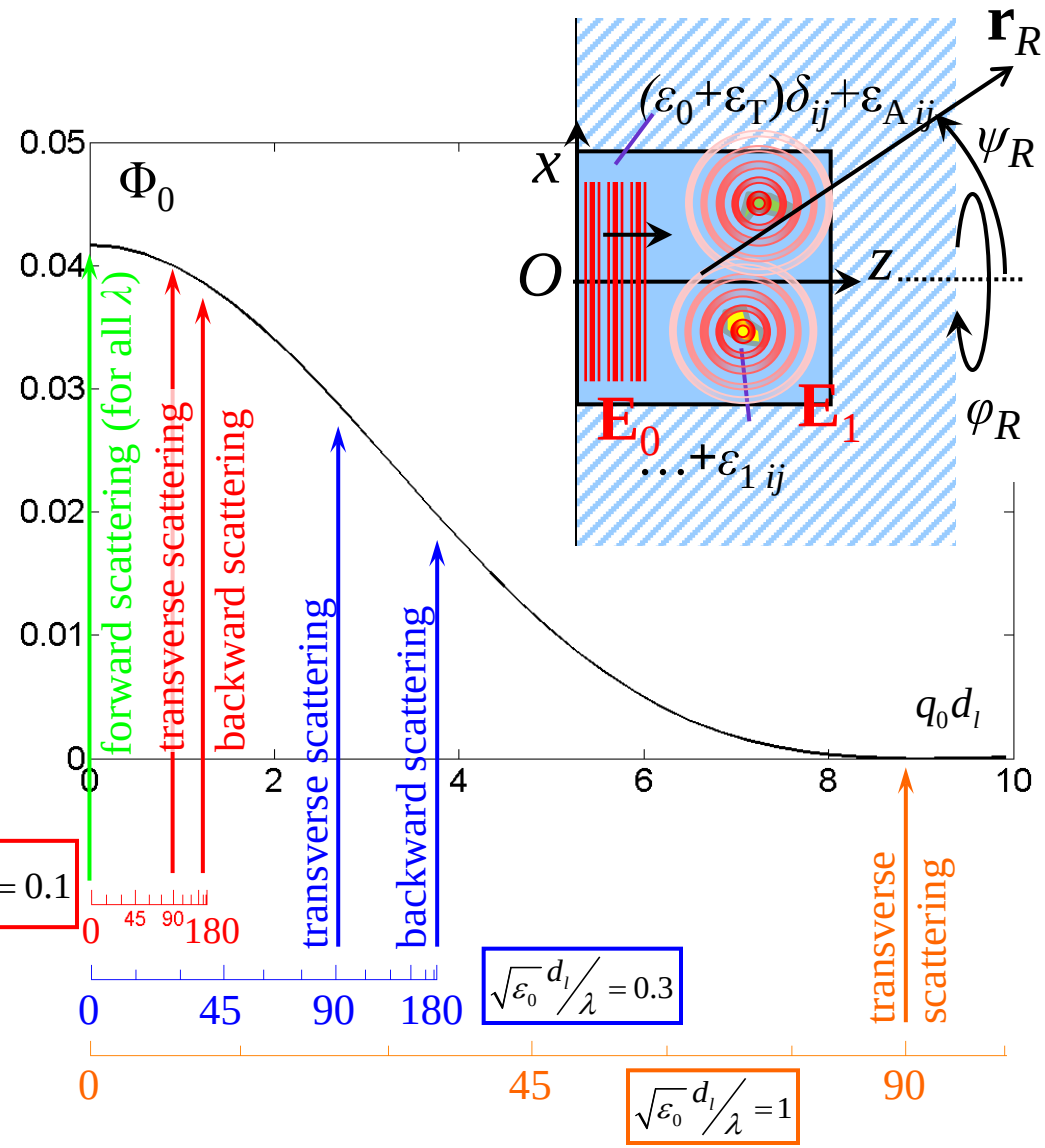
In free space:





corresponding ψ angles for various wavelengths

The scattered power fraction

$$\langle Q\rangle = (\Lambda p)^2 (d_l/L)(\xi-1)^2 \left[3I_1(\Lambda) - 2I_2(\Lambda)\right]/35$$

$$\Lambda = k_0 d_l \sqrt{\varepsilon_0}$$

$$\Lambda/2\pi = \sqrt{\varepsilon_0}\, d_l/\lambda$$

$$\xi = \frac{\pi_{44}}{\pi_{11} - \pi_{12}}$$ is the photoelastic anisotropy parameter

$\Lambda << 1$:

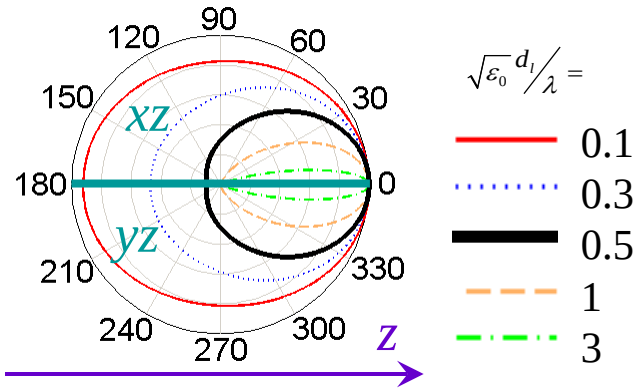
$I_1 \approx 1/144$, $I_2 \approx 1/864$

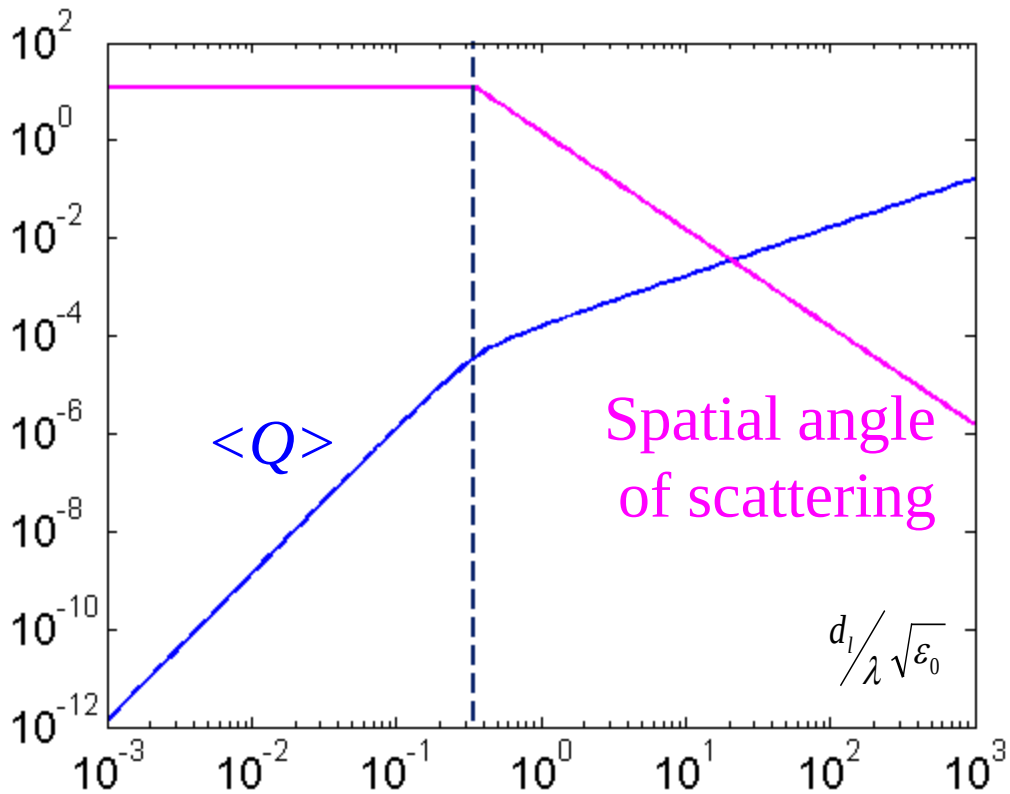
$\langle Q\rangle \approx (\Lambda p)^2(\xi-1)^2 d_l/(1890L)$

$\Lambda >> 1$:

$I_1 \approx \Lambda^{-2}/32 >> I_2$

$\langle Q\rangle \approx p^2(\xi-1)^2 d_l/(1130L)$





(YAG, L = 1 cm, p ≈ 15)

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Comparison of the results with the geometric optics calculation

- The scattered power fraction

$$\langle Q \rangle = (\Lambda p)^2 (\xi - 1)^2 \frac{d_l}{L} \frac{3I_1(\Lambda) - 2I_2(\Lambda)}{35}$$

The isotropic approximation

$$\varepsilon = \frac{\varepsilon_{rr} + \varepsilon_{\varphi\varphi}}{2}$$

$$\langle Q_{\text{iso}} \rangle = (\Lambda p)^2 (\xi - 1)^2 \frac{d_l}{L} \frac{3I_1 - 6I_2}{350}$$

$\Lambda >> 1$:

$I_1 \approx \Lambda^{-2}/32 >> I_2$

$<Q> \approx 3p^2(\xi-1)^2 d_l/(1120L)$

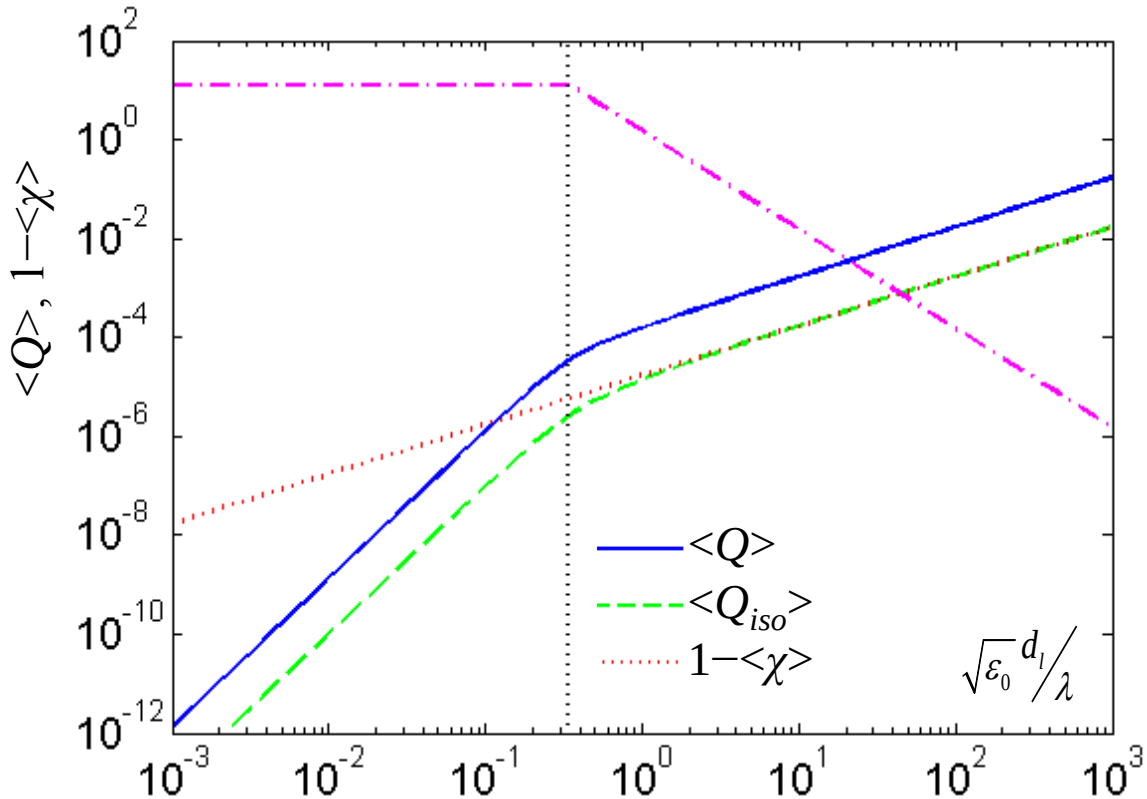
$<Q_{\text{iso}}> \approx 3p^2(\xi-1)^2 d_l/(11200L)$

- The fundamental mode power loss in the geometrical optics approximation (+ the isotropic approximation)

$$1 - \langle \chi \rangle = \frac{3p^2(\xi - 1)^2}{11200} \frac{d_l}{L}$$

[1] Snetkov, I. L., Mukhin, I. B., Palashov, O. V., and Khazanov, E. A., Quantum Electron. 37(7) 633–638 (2007).

Comparison of the results with the geometric optics calculation



(YAG, L = 1 cm, λ = 1030 nm, $p \approx 15$)

Conclusion

- Thermally induced light scattering in laser ceramics has been investigated for an unconditioned grain size to wavelength ratio using the Debye-Rayleigh scattering theory in single scattering approximation.
- A modeling spatial spectrum of the dielectric permeability variations in ceramics was suggested.
- The directional pattern of scattered radiation, the extinction coefficient, and the scattered power fraction were calculated.
- In the case of large grains the directional pattern is narrow and the scattered power is proportional to the grain size. In the opposite case scattering is almost isotropic, and the scattered power is proportional to the grain size cubed, therefore rapidly lessens.
- In the large grain case the scattered power fraction was compared with the ratio of the fundamental mode radiation loss calculated in the geometric optics approximation. The values obtained coincide.

Thank you for your attention